Engineering Notes

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Impulsive Motion of a Cylinder and Viscous Cross Flow

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COMPUTER programs[1] for determining the forces and moments on aerodynamic bodies used in computer-aided design procedures are limited to flowfields of low angles of attack since available aerodynamic theory, primarily inviscid theory, is restricted to this range. Extensions of theory to higher angles of attack must necessarily take viscous effects into account in some manner due to boundary-layer separation. For a slender body of revolution in subsonic or supersonic flow at moderate angle of attack, use has been made of the analogy between the viscous cross flow and the two-dimensional, time-dependent, viscous, incompressible flow induced by a circular cylinder in impulsive motion.[2,3]

This idea has been employed in various ways. Two characteristics of the methods have been the use of experimental data for the two-dimensional time-dependent flow and the empirical fitting of this data to the pertinent class of three-dimensional flowfields.

It is the purpose of this Note to offer a theoretical solution for the problem of an impulsive start of a circular cylinder (which must be of higher order in $Re^{-1/2}$ than in classical boundary-layer theory to supply viscous effects) and to test its applicability as a description of the viscous cross flow of a three-dimensional body. A particular case of the latter, that of an ogive cylinder at $M = 2$ (Ref. 4), is chosen as a test case.

1.0 Impulsive Motion of Cylinder

The flowfield induced by the impulsive motion of a cylinder is given as a solution to the following problem: for axes fixed in the cylinder and normalizing the variables using cylinder radius a, freestream velocity V, and a characteristic time t_0, arbitrary at this point,

$$\text{DE } \omega_{,t} - \delta^2\Delta\omega = -(\epsilon/r)\partial(\psi,\omega)/(r,\theta);$$
$$\omega = \Delta\psi = \psi_{,rr} + \psi_{,r}/r + \psi_{,\theta\theta}/r^2$$
$$\text{IC } t = 0: \quad \psi = 0 \tag{1}$$
$$\text{BC } r = 1: \quad \psi = \psi_{,r} = 0, r \to \infty: \quad \psi \to r\sin\theta$$

where $\delta^2 = \nu t_0/a^2 \ll 1$, $\epsilon = Vt_0/a \ll 1$ (such that $Re = \epsilon/\delta^2 = Va/\nu$).

From the momentum equation

$$p(1,\theta,t) = \frac{2}{Re}\int_0^\theta \omega_{,r}|_{r=1}\,d\theta + p_0(t) \tag{2}$$

where p has been normalized by $\rho V^2/2$ and $p_0(t) = p(1,0,t)$.

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A solution is sought in terms of the two small parameters δ and ϵ. The expansion in ϵ is a regular parameter perturbation with $t = t^*/t_0 \sim 0(1)$ (t^* being dimensional time) which is equivalent to a coordinate expansion in $t = t^*/t_0 \ll 1$. The expansion in δ is a singular perturbation and so asymptotic inner and outer solutions are required in this parameter. Thus, a solution is sought in composite additive form as

$$\psi(r,\theta,t;\delta,\epsilon) \cong \sum_{k=0}^{\Gamma-2}\delta^k\Psi_k(r,\theta,t;\epsilon) + \left\{\delta\sum_{k=0}^{\Gamma-2}\delta^k\psi_k(\bar{r},\theta,t;\epsilon) - \langle CP\rangle\right\} + 0(\delta^\Gamma) \cong \Psi_0 + \delta\Psi_1 + \delta\{\psi_0 - [\bar{r}\psi_{0,r} - \psi_1]_{r=1}\} + \delta^2\{\psi_1 - [(\bar{r}^2/2)\psi_{0,rr} - \bar{r}\psi_{1,r}]_{r=1}\} + 0(\delta^3) \tag{3}$$

where $\bar{r} = (r-1)/\delta$ is the inner variable, $0(\delta^\Gamma)$ is the error in the neighborhood of $r = 1$, $\langle CP\rangle$ is the common part of the two expansions, and

$$\lim_{\bar{r}\to\infty}\{\quad\} = 0$$

exponentially, the latter supplying the missing boundary conditions in the inner and outer problems.

Substitution of the outer expansion $\Sigma\delta^k\Psi_k$ into Eqs. (1) and taking outer limits yields a set of outer problems with BC on the surface relaxed (except for $\Psi_0|_{r=1} = 0$) and substituting the inner expansion $\Sigma\delta^{k+1}\psi_k$ into Eqs. (1) in the inner variable $\bar{r}$ and taking inner limits yields a set of inner problems with the BC at ∞ relaxed.

Then each equation (except that for Ψ_0) involves the small parameter ϵ. Solutions to each of these problems is sought in expansions of ϵ. It is noted that both, δ, a normalized boundary-layer thickness and ϵ, normalized time elapsed, grow with time. Thus, the expansion in ϵ is a finite one. It can be seen that the inner expansion controls. Thus, let

$$\psi_n(\bar{r},\theta,t;\epsilon) = \sum_{\beta=0}^{\sigma_k}\epsilon^\beta\psi_{k\beta}(\bar{r},\theta,t) \tag{4}$$

where $\sigma_k < (1 - \log Re/\epsilon)(\Gamma - k - 1)/2 < \sigma_k + 1$.

This is similar to Wang[5] but retains the distinction between δ as the parameter in the singular perturbation and ϵ as the parameter in the regular perturbation.

Following the previous procedure,

$$\Psi_0(r,\theta,t) = H(t)[q(\theta)/2][r - 1/r] \tag{5}$$

where $q(\theta) = 2\sin\theta$ and $H(t) = 0, t \leq 0; = 1, t > 0$, is the potential flow about the cylinder, valid for small time.

Assuming

$$\psi_0(\bar{r},\theta,t;\epsilon) = 2q\cdot(t)^{1/2}\{\zeta_{00}(\eta) + \epsilon t q'\zeta_{01b}(\eta) + (\epsilon t)^2[q'^2\zeta_{02a}(\eta) + qq''\zeta_{02b}(\eta)] + \ldots\} \tag{6}$$

where $\eta = \bar{r}/2(t)^{1/2}$, one obtains the ordinary differential equations for the ζ functions as in Wundt[6] [Eqs. (14–18)]. The values of these functions and their derivatives are tabulated therein.

Continuing

$$\Psi_1(r,\theta,t;\epsilon) = 2q(t)^{1/2}(-1/r(\pi)^{1/2} + \epsilon t q'\zeta_{01}(\infty)/r^2 + (\epsilon t)^2\{(\zeta_{02a}(\infty) - 3\zeta_{02b}(\infty))/r + [\zeta_{02a}(\infty) + \zeta_{02b}(\infty)] \times (3 - q^2)/r^3\} + \ldots) \tag{7}$$

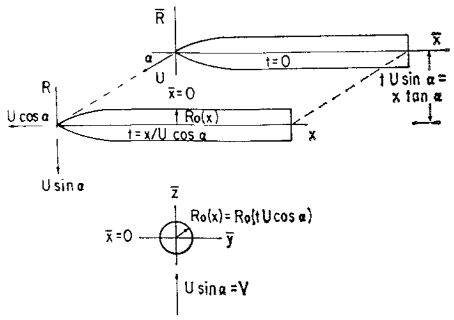

Fig. 1. The analogy.

which is a potential flow displaced by the thickness of the boundary layer.

If ψ_1 is available, then the pressure on the surface is given from Eq. (2) as

$$p(1,\theta,t) = 2\int_0^\theta [\psi_{0,\bar{r}\bar{r}\bar{r}}/\epsilon + (\psi_{1,\bar{r}\bar{r}\bar{r}} + \psi_{0,\bar{r}\bar{r}})/(\epsilon Re)^{1/2}]_{\bar{r}=0} d\theta + p_0(t) + 0(1/Re) \quad (8)$$

Now ψ_0 is given in Eq. (6) and only $\psi_{1,\bar{r}\bar{r}\bar{r}}|_{\bar{r}=0}$ is required. The latter is obtainable from the equation for ψ_1 itself (i.e., no integration is required). With this, the coefficient of pressure $C_p = p - p_\infty = p - p_0 + 1 + 0(1/Re)$ is

$$C_{p2} = 1 - 4\sin^2\theta + 1/(Re)^{1/2}\{A_0(1 - \cos\theta)/(\epsilon t)^{1/2} + A_1 \sin^2\theta\cdot(\epsilon t)^{1/2} + (\epsilon t)^{3/2}[A_{20} + A_{21}\cos\theta + A_{23}\cos^3\theta] + \ldots\} + 0[\epsilon^2(\epsilon/Re)^{1/2}, 1/Re] \quad (9)$$

where $Re = Va/\nu$ and $\epsilon t = Vt^*/a$ and $A_0 = -2.256758$, $A_1 = 13.815280$, $A_{20} = -8.028938$, $A_{21} = -5.720120$, and $A_{23} = 13.749012$.

Note that $1 - 4\sin^2\theta$ is the potential flow expression which arises from the first term in Eq. (8) given by Wundt.[6] Thus, the higher-order term involving ψ_1 is required for the viscous contribution.

Note also, that extension of Eq. (9) to higher orders in ϵ and $Re^{-1/2}$ is readily available through numerical integration of the ordinary differential equations implied by the previous solution procedure.

2.0 Viscous Cross-Flow Analogy

The essence of the analogy between the viscous cross flow of the three-dimensional flow and the two-dimensional time-dependent flow is contained in Fig. 1 for a body of revolution at angle of attack.

An observer in a lamina $\bar{x} = 0((\bar{x},\bar{y},\bar{z})$ fixed in the free-stream) sees a two-dimensional flow over a cylinder of varying radius. This flowfield is approximated by a circular cylinder of constant diameter impulsively started from rest. The pressure distribution of this latter flowfield, which is a function of t where t is correlated with the distance along the body as $t = x/U\cos\alpha$, is assumed to be additive with the pressure distribution given by potential theory for a body of revolution at angle of attack.

Thus, the analogy implies that the viscous contribution of the two-dimensional time-dependent flow

$$\Delta C_{p2v} = C_{p2}(x,\theta) - (1 - 4\sin^2\theta) \quad (10)$$

C_{p2} given by Eq. (9), is to be equal to the nonlinear aerodynamic or viscous cross-flow effect,

$$\Delta C_p(x,\theta) = C_{p,\mathrm{exp}} - C_{p,\mathrm{pot}} \quad (11)$$

where $C_{p,\mathrm{exp}}$ is the experimentally measured pressure distribution and $C_{p,\mathrm{pot}}$ is that given by potential theory.

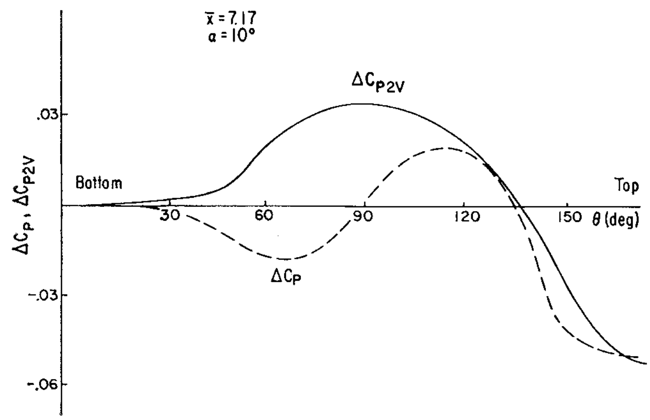

Fig. 2 Theoretical and experimental viscous effects.

To supply the correlation between the two- and three-dimensional flow let

$$Re = U\sin\alpha R_0^*(x)/\nu; \quad \epsilon t = Vt^*/R_0^*(x) = x^*\tan\alpha/R_0^*(x) \quad (12)$$

in Eq. (9), where $(\ )^*$ are dimensional quantities. Thus, instantaneous values of the radius are used.

3.0 Discussion

In applying Eqs. (9) and (10) as a description of the viscous cross flow of a three-dimensional body, for example the ogive-cylinder body of Fig. (1),[4] the first question raised is the correspondence between the solution for a constant diameter cylinder and the solution for a cylinder of varying radius implied by the ogival nose. In the former, infinite pressures are induced by the impulsive start and the process leading to separation begins immediately. Separation then occurs at about $\epsilon t = \frac{1}{3}$. For the two-dimensional counterpart of the ogival nose, there are no infinite pressures and separation is delayed due to the increasing radius. To establish the correspondence, ignore the term $\sim(\epsilon t)^{-1/2}$ in Eq. (9) and match the two solutions at the point of separation; i.e., replace ϵt in Eq. (11) by

$$\hat{\epsilon t} = \tfrac{1}{3} + [x - x_s(\alpha)]\cdot 2\tan\alpha \quad (13)$$

where $x = x^*/D$ is the diameter of the cylindrical portion, and $x_s(\alpha)$ is the station at which vortices begin to separate from the three-dimensional body, experimentally determined in Ref. 4. For $x < x_s(\alpha)$, potential theory suffices within this approximation.

Now, since Eq. (9) is a small time solution, it is confined to small angles of attack or large fineness ratio (i.e., $\epsilon t < 1$) or, in the case of slender bodies, to a limited distance downstream of $x_s(\alpha)$.

Applying Eqs. (9–13) to the ogive cylinder at $M = 2.0$, at $\alpha = 10°$, for which $x_s = 6.0$, at station $x = 7.17$, yields a ΔC_{2pv} which is compared with the $\Delta C_p(\theta)$ given by Eq. (12) ($C_{p,\mathrm{pot}}$ given by Van Dyke's hybrid theory). These are plotted in Fig. 2. The error in ΔC_p is approximately ± 0.0075, while the error in ΔC_{p2v}, given by the error term in Eq. (9), is of the order of ± 0.001. The error in the choice of x_s is roughly ± 0.01.

A comparison was also made for the classical case of a spheroid ($FR = 4$) in incompressible flow for which the exact potential solution is known.[7] In this case, there is a larger discrepancy between the two-dimensional constant diameter solution and the cylinder of time-varying radius implied by the spheroid. Ignoring the term $\sim(\epsilon t)^{-1/2}$ and shifting the two-dimensional solution to the station of maximum diameter [i.e., let ϵt, Eq. (9), be $(x - FR/2)\cdot(2\tan\alpha)D(x)$, $x = x^*/\max D(x)$, gave a ΔC_{p2v} within an order of magnitude of ΔC_p for $0.5\,FR < x < 0.9FR$.

Thus, the theoretical two-dimensional time-dependent solution, Eq. (9), has been tested as a description of the viscous cross flow for large fineness ratio bodies approximating circular cylinders at moderate angles of attack. Some agreement has been obtained, although empirical procedures are still required and the region of applicability is narrow. For slender bodies of high angles of attack, where vortices depart an appreciable distance from the body surface, Eq. (9), a boundary-layer-type solution, would be invalid.

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Parametric Shimmy of a Nosegear

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Nomenclature

a = a parameter in a standard Mathieu Equation (15)
a_0, a_2 = coefficients of the characteristic Eq. (4) when $\epsilon = 0$
C = the coefficient of yaw of a tire (rad/lb)
f = force generated by the wheel and tire imperfections, in the wheel plane (lb)
F_n = lateral ground reaction (lb)
F_r = vertical ground reaction (lb)
g = a parameter defined in Eq. (16)
H = distance from trunion to axle (in.)
I_f = moment of inertia of the system about the strut axis (lb-in.-sec²)
I_s = moment of inertia of the swivelling parts about the spindle (lb-in.-sec²)
K_0 = torsional spring constant about the strut axis (in.-lb/rad)
K_t = torsional spring constant about the spindle (in.-lb/rad)
L = trail length (in.)
m = wheel mass (lb-sec²/in.)
M_f = the moment of the force f about the strut axis (in.-lb)
q = a parameter in a standard Mathieu Equation (15)
R = wheel radius (in.)
t = time variable (sec)
u = the amount of unbalances (in.-oz)
V = vehicle velocity (in./sec or mph)
W = weight on the nosegear (lb)
α = the camber angle of the nosegear (rad)
$\epsilon, \epsilon_1, \epsilon_2$ = the product of the force f and the trail length, refer to Eqs. (5) and (12)
θ = the yaw angle of the nosegear (rad)
τ = nondimensional time variable defined in Eq. (13)
ω = the frequency of rotation of the wheel (rad/sec)
ω_1, ω_2 = the fundamental frequencies of the nosegear (rad/sec)

Symbols

[ ] = a square matrix
{ } = a column matrix
$\dot{x}$ = dx/dt

In the previous studies[1–3] of the nosegear shimmy, the effect of wheel and tire imperfections were not discussed. The purpose of this Note is to show that instability of certain nosegear can result from large wheel and tire imperfections. And the phenomenon of this self-excited instability (shimmy) is similar to that of the well-known parametric resonance.[4,5] It is different from the ordinary resonance caused by an external periodic force in that the major instability occurs when the wheel rotation frequency is equal to double any natural frequency of the system. For a simplified nosegear system studied in this Note, we find this parametric shimmy of landing gear is determined completely by the instability character of an associated Mathieu equation.

To avoid great mathematical difficulties, we use a much simplified nosegear model shown in Fig. 1. This model resembles essentially that used by Moreland,[2] and has two degrees of freedom (camber α and yaw θ). In this study, the forces generated by all wheel and tire imperfections, e.g., unbalances, flat spots, etc. are lumped together and represented by a single force f which is the centrifugal force generated by a single unbalance weight located at the periphery of the tire Fig. (1b) thus,

$$f(\omega) \approx (u/6176)\omega^2, \; \omega \approx V/R \qquad (1)$$

The quantities in the previous and subsequent equations are defined in the nomenclature. Note in the yawed position

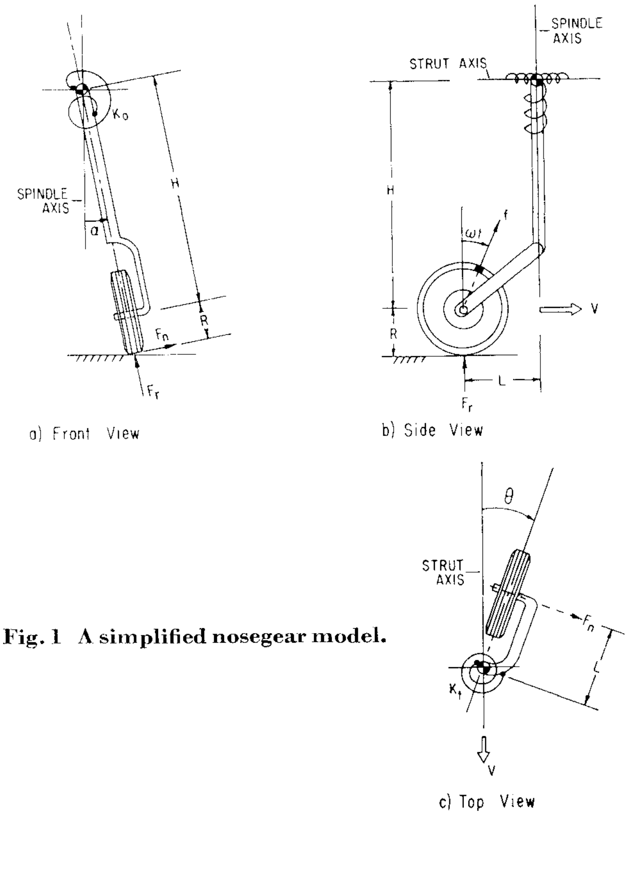

Fig. 1 A simplified nosegear model.

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